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RELATIVISTIC SELF-DUAL CHERN-SIMONS SYSTEMS: A PERSPECTIVE*

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The self-dual systems are constrained and so are simpler to understand. In recent years there have been several studies on the self-dual Chern-Simons systems. Here I present a brief survey of works done by my collaborators and myself. I also discuss several questions related to these self-dual models.

1. Introduction

In three dimensional spacetime, besides the Maxwell term, the parity-violating Chern-Simons term can be the kinetic part for the gauge field.[1,2] The Chern-Simons Lagrangian for an abelian gauge field A_μ is given as

$$\mathcal{L}_{CS} = \frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho, \tag{1}$$

and the corresponding term for a nonabelian gauge field A^a_μ is

$$\mathcal{L}'_{CS} = \frac{\kappa}{2}\epsilon^{\mu\nu\rho}(A^a_\mu\partial_\nu A^a_\rho + \frac{1}{3}f^{abc}A^a_\mu A^b_\nu A^c_\rho), \tag{2}$$

where the coefficients f^{abc} are the structure constants of the gauge group. The Lagrangians (1) and (2) are invariant under infinitesimal gauge transformations which vanish at spatial infinity. For quantum amplitude $\exp(i\int d^3x\mathcal{L}'_{CS})$ to be invariant under large gauge transformations, the coefficient κ of Eq. (2) should be quantized.[2]

The abelian theory of a complex scalar field ϕ coupled to A_μ with the Chern-Simons kinetic term is defined by the Lagrangian

$$\mathcal{L}_1 = \mathcal{L}_{CS} + D_\mu\phi^\dagger D^\mu\phi - U(\phi), \tag{3}$$

where $D_\mu\phi = (\partial_\mu - iA_\mu)\phi$. We will consider the case of the pure Chern-Simons kinetic term as the Maxwell term, if present additionally, will affect only the short distance physics. The Gauss law of the theory (3) is

$$\kappa F_{12} = J_0, \tag{4}$$

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where $J_\mu = i(D_\mu \phi^\dagger \phi - \phi^\dagger D_\mu \phi)$. Thus total magnetic flux $\Psi = \int d^2x\, F_{12}$ is related to total charge $Q = \int d^2x\, J_0$ by $\kappa\Psi = Q$. The basic excitations of the system are either charge neutral particles or charge-flux composites.

The conserved angular momentum for the Lagrangian (3) is

$$J = -\int d^2x\, \epsilon^{ij} x^i \left(D_0\phi^* D_j\phi + D_j\phi^* D_0\phi\right), \tag{5}$$

whose density is gauge-invariant and localized. Under the CTP symmetry the sign of the angular momentum does not change, and so particles and antiparticles carry the same spin. In the symmetric phase particles carry nonzero spin $1/(4\pi\kappa)$. In the broken phase there are elementary neutral scalar and vector bosons, and also charged magnetic flux vortices whose spin is $-\pi\kappa$. Note the sign difference of anyon spins in the symmetric and broken phases.

In two dimensional space particles of fractional spin, anyons, are possibilities.[3] In the symmetric phase of the Chern-Simons-Higgs systems, anyons are represented by charge-flux composites. The fractional statistics can be understood by considering the orbital angular momentum of a pair of anyons or anyon-antianyon interacting under a central force. In the system of two identical anyons of spin s, the allowed orbital angular momentum is $L = 2l + 2s$ with an integer l and so the total angular momentum is $J = L + 2s = 2l + 4s$. For the system of anyon-antianyon, the allowed orbital angular momentum is $L = 2l - 2s$ so that the total angular momentum is $J = L + 2s = 2l$. This makes possible to creat pairs of anyon and antianyon by vacuum fluctuations. The fractional statistics arise when we gauge away the flux carried by charged bosons. For the detail of anyon physics, the readers can consult many review articles.[4]

There are already a few reviews on the Chern-Simons Higgs systems.[5,6] This talk is a brief survey of the relativistic Chern-Simons systems, focusing on work done by my collaborators and myself. In Sec. 2, I summerize the salient features of the self-dual abelian Chern-Simons-Higgs model. In Sec. 3, the self-dual models with nonabelian gauge symmetry are discussed. In Sec. 4, the self-duality is generalized to the sigma and $CP(N)$ models. In Sec. 5, some general ideas, like supersymmetry, the correction to the Chern-Simons coefficient, and moduli space approximation of low energy dynamics of vortices, are discussed. Here I summerize some questions whose answer seems not known. Section 6 contains concluding remarks.

2. Self-dual Abelian Chern-Simons Higgs Systems

One of the first self-dual models found is the self-dual Maxwell-Higgs system.[7] The relativistic self-dual Chern-Simons-Higgs system has been found later on.[8] The self-dual Lagrangian is given by Eq.(3) with a specific potential

$$U(\phi) = \frac{1}{\kappa^2}|\phi|^2(|\phi|^2 - v^2)^2. \tag{6}$$

The theory is renormalizable and the only dimensionful parameter is v^2. When v^2 vanishes, there is a classical scaling symmetry which may be broken quantum

mechanically. With the help of the Gauss law (4), the energy of the model can be expressed as

$$E = \int d^2x \left\{ |D_0\phi \pm \frac{i}{\kappa}(|\phi|^2 - v^2)\phi|^2 + |D_1\phi \pm iD_2\phi|^2 \right\} \pm v^2\Psi, \tag{7}$$

where there is no boundary contribution as we consider only finite energy configurations. The Bogomolny bound on the energy is then

$$E \geq \pm\frac{v^2}{\kappa}Q. \tag{8}$$

This bound is saturated by configurations satisfying the self-dual equations,

$$D_0\phi \pm \frac{i}{\kappa}(|\phi|^2 - v^2)\phi = 0, \qquad D_1\phi \pm iD_2\phi = 0. \tag{9}$$

The above equations imply that $\partial_0|\phi| = 0$ and so the field configuration can be static in time in a given gauge choice. Combined with the Gauss law (4), the self-dual equations can be put to

$$\partial_i^2 \ln|\phi|^2 - 4|\phi|^2(|\phi|^2 - v^2) = 4\pi\sum_a \delta^2(x^i - q_a^i), \tag{10}$$

where q_a^i's are the positions of vortices.

The potential has two degenerate minima; the symmetric phase where $<\phi>= 0$ and the broken phase where $<\phi>= v$. As mentioned before, there are elementary excitations in both phases and self-dual anyonic vortices in the broken phase. In the symmetric phase there are also self-dual anyonic nontopological solitons of unquantized magnetic flux.[9]

In the broken phase the self-dual configurations are determined unquely by vortex positions.[10] While the energy is degenerate, the angular momentum is a complicated function of vortex positions.[11] The statistics of anyons in the symmetric phase is decided by the Aharonov-Bohm phase. The statistics of anyonic vortices in the broken phase have the contributions from both the Aharonov-Bohm phase and a phase originated from the Magnus force. These two phases can be combined into a single dual phase in the dual formalism where vortices appear as charged elementary particles, explaining the fore-mentioned sign difference of anyon spins.[11,12]

The nonrelativistic limit of this self-dual model has been found and studied.[13] The self-dual systems with the both Maxwell and Chern-Simons kinetic terms have been also found.[14] This self-dual model interpolates smoothly between the Maxwell-Higgs and Chern-Simons-Higgs systems.

A further generalization of these self-dual models by including unform background charge has been found.[15] The whole structure of this model is quite rich. Some phase appears to be infinitely degenerate, some self-dual solitons have a negative rest mass even though their kinetic mass is positive, in some phase there is a roton mode among elementary excitations, some phase appears to be inhomogeneous, implying spontaneous breaking of translation symmetry, etc. The definition

of the angular momentum becomes delicate as in the Maxwell-Higgs case with the background charge.[16]

3. Self-dual Nonabelian Chern-Simons-Higgs Systems

The previous abelian self-dual model can be generalized to the self-dual systems with nonabelian gauge group.[17,18] The crucial point is to require that there exists at least a global $U(1)$ symmetry. For simplicity we consider the theory of a complex scalar field ϕ in a given irreducible representation of the gauge group. If the representation is real, we need two real scalar fields to make the matter field complex. The generators of the symmetry group in this representation are hermitian matrices T^a. The conserved global $U(1)$ symmetry is generated by a global phase rotation, $\phi \rightarrow e^{i\alpha}\phi$. The self-dual Lagrangian is then

$$\mathcal{L}_2 = \mathcal{L}'_{CS} + |D_\mu \phi|^2 - \frac{1}{\kappa^2}|T^a\phi\phi^\dagger T^a\phi - v^2\phi|^2, \tag{11}$$

where $D_\mu\phi = (\partial_\mu - iT^a A^a_\mu)\phi$. The energy bound is given by Eq.(8) with the global charge $Q = i(D_0\phi^+\phi - \phi^\dagger D_0\phi)$.

Similar self-dual models with the pure Yang-Mills kinetic term are possible. However, there seem no interesting solitons here. These models may be regarded as a bosonic part of theories with an extended supersymmetry.

Interesting vacuum and soliton structures show up when the matter field is in adjoint representation. The vacuum expectation value of the potential satisfies the algebraic equation

$$[[\phi, \phi^\dagger], \phi] = v^2\phi, \tag{12}$$

where $\phi = \phi^a T^a$. This equation is the $SU(2)$ Lie algebra with identification $J_3 = [\phi, \phi^\dagger]/v^2$ and $J_+ = \phi/v$. This allows the detail analysis of vacuum and mass spectrum.[19,20] The solitonic structure in the $SU(3)$ case has been studied in detail.[19]

The nonrelativistic limit of this theory represents a theory of anyons with nonabelian statistical phase. There are extensive work and review of self-dual solitons in this limit.[21,6] The dynamics of vortices in the broken phase may involve the nonabelian generalization of the Magnus force.

4. Sigma and $CP(N)$ Models

The sigma model has been studied extensively, where self-duality has also been explored.[22] The self-dual field configurations are topological lumps which are characterized by the second homotopy of the field as a mapping from two dimensional space to the internal field space. By gauging a part of the global symmetry of the sigma model, one can have another self-duality. Especially a new self-dual sigma model with the Maxwell term has been found recently.[23] This has been further generalized to the models with the Chern-Simons term.[24,25]

These models have been generalized further to the $CP(N)$ models where the matter field is a complex vector field $z = (z_1, ..., z_{N+1})$ of unit length. The nontrivial

generalization is achieved by gauging the part of the global $SU(N+1)$ symmetry such that there exists at least one conserved global $U(1)$ group which commutes with the gauge group.[26] With the gauge symmetry generators T^a and the global symmetry generator R, the covariant derivative is

$$\nabla_\mu z = (\partial_\mu - iT^a A^a)z - z(\bar{z}(\partial_\mu - iT^a A^a)z), \tag{13}$$

and the Lagrangian is

$$\mathcal{L}_{CP(N)} = \mathcal{L}'_{CS} + |\nabla_\mu z|^2 - \frac{1}{\kappa^2}\left|\big(T^a z - z(\bar{z}T^a z)\big)\bar{z}T^a z - v\big(Rz - z(\bar{z}Rz)\big)\right|^2. \tag{14}$$

The conserved topological current is then $K^\mu = -i\epsilon^{\mu\nu\rho}\partial_\nu(\bar{z}(\partial_\rho - iA_\rho)z)$. The global current for R is $J^\mu = i(\nabla^\mu \bar{z}(Rz - z(\bar{z}Rz)) - h.c.)$. The Bogomolny bound is given by $E \geq |T|$ where $T = \int d^2x(K^0 + vJ^0/\kappa)$.

In certain limits the self-dual Chern-Simons $CP(N)$ models approach all known self-dual Chern-Simons Higgs models, implying that the $CP(N)$ models have all the complicated vacuum and soliton structures as the Higgs cases, and more. Especially with the matter in adjoint representation, the vacuum condition is identical to Eq.(12) for an appropriate range of v. The structure of the self-dual configurations is not yet fully explored.

5. General Ideas and Questions

The self-dual Chern-Simons-Higgs systems are renormalizable while the self-dual $CP(N)$ models are not. We can use the perturbative approach to calculate the quantum effect in the Higgs case, which has not been fully explored even at one-loop. When one introduces uniform background charge, the theory is still renormalizable as far as the dimensional counting is concerned. As the Lorentz symmetry is lost in this case, there may be some surprises.

In the abelian case uniform external charge can be introduced and is neutralized by the Higgs charge in the broken phase. One can ask whether there exists a similar configuration in nonabelian self-dual systems. I think that it does because one can imagine a configuration where the global $U(1)$ charge is distributed unformly, exactly like Q-matters.[27]

There is also a curious homogeneous configuration whose properties are not fully explored. The vector potential here is uniformly rotating; $(A_1 + iA_2) = ce^{iwt}$. The energy density is homogeneous and constant parameters c, w are determined by the field equations.[28] We do not know whether such configuration is classically or quantum mechanically stable. This case may be somewhat analogous to the uniform current case in the Maxwell-Higgs case where the current decays by nucleating vortex loops.[29]

5.1. *Supersymmetry*

For every self-dual model we expect that there exits an underlying supersymmetry.[30] The $N = 2$ supersymmetric models behind the self-dual Chern-Simons-Higgs system have been found.[31] The central charge of the $N = 2$ supersymmetry gives the Bogomolny bound on the Hamiltonian. In the broken phase, the supermultiplet of massive neutral vector bosons can have the spin structure $\pm(1, 1/2, 0, -1/2)$ at most as there is only one degree of freedom associated with vector bosons. Thus the maximal supersymmetry allowed in this models is $N = 3$.[32] When there is uniform background charge, the supersymmetry is not obvious at all as the mass spectrum of a given supermultiplet does not show the degeneracy.[15,16]

In the $N = 2$ or $N = 3$ supersymmetric cases the Bogomolny bound is expected to be exact because the charged sector saturating the energy bound has a reduced representation and the supersymmetry is not supposed to be broken. The $N = 2$ supersymmetric theories needs the infinite renormalization.[33,34] On the other hand the $N = 3$ supersymmetric theories seem to be finite at least one loop.[35,36] It would be interesting to find out whether the $N = 3$ models are finite in all orders.

When the parameter v vanishes, the classical field theory has the scaling symmetry, which may be broken quantum mechanically by the Coleman-Weinberg mechanism.[37] Indeed recently such mechanism is shown to work here by calculating two-loop diagrams.[38] The scale symmetry may be preserved quantum mechanically for the $N = 2, 3$ supersymmetric models. If this is the case, these supersymmetric theories have a quantum superconformal symmetry.

Recently there has been a considerable progress in understanding of the low energy nature of the self-dual Yang-Mills Higgs systems with the $N = 4$ supersymmetry in three dimensions.[39] Similar to these systems, in Chern-Simons-Higgs systems magnetic monopole instantons exist.[40] It would be interesting if one can make similar exact statements for the $N = 2, 3$ supersymmetric Chern-Simons-Higgs systems.

Following an argument similar to that for getting $N = 3$ for the maximally supersymmetric Chern-Simons-Higgs systems, one can see the maximal supergravity theory with massive gravitons should be $N = 7$. It would be interesting to see whether this theory, if constructed, is finite.

5.2. *Chern-Simons Coefficient in the Broken Phase*

In the abelian Chern-Simons theories the Coleman-Hill theorem states that the Chern-Simons coefficient does not get corrected except by the fermion contribution at one loop when the gauge symmetry is not spontaneously broken and there is no massless charged particle.[41] The vacuum polarization by the fermion loop renormalizes the bare Chern-Simons coefficient at the scale larger than the fermion Compton length. When the gauge symmetry is partially broken, the correction to the coefficient for the unbroken gauge group is shown to be still quantized[42].

This theorem has been extended to the broken phase, where the 'total' Chern-

Simons term is argued to be a sum of the 'pure' renormalized Chern-Simons term plus an 'effective' term involving the scalar field which looks like the Chern-Simons coefficient.[43] One-loop correction to the pure coefficient is quantized. That to the effective coefficient is not quantized in general. In the self-dual abelian Chern-Simons-Higgs system, the correction to the effective term is however quantized.[35] This may be true even with nonabelian gauge symmetry with the pure Chern-Simons kinetic term. It would also be interesting to find out whether there is a quantum correction to the vortex spin and if it does, whether it is related to the correction to the coefficient.

Suppose that many family of bosons become massive fermions by a Chern-Simons interaction and they are coupled to another gauge field. The natural question is whether these composite fermions induce the Chern-Simons term to another gauge field. If it does, the composite fermions can be treated as fundamental fermions.

5.3. *Low Energy Dynamics of Vortices*

In the broken phase of the theory considered in Sec.2, the self-dual configurations for n vortices, with gauge equivalent configurations identified, form a finite dimensional moduli space. The natural coordinates for this moduli space are the vortex positions $q^i_a, a = 1,,,n$. One expects the low energy dynamics of these vortices can be described as dynamics on the moduli space.[44] There is no potential energy for vortices as the energy is degenerate. However there exists a term linear in velocities as the total angular momentum depends on vortex positions.[11] This linear term leads to the statistical interaction between vortices and is originated from the sum of the naive gauge interaction and the Magnus force.[11] The most general nonrelativistic Lagrangian for the moduli coordinates is then

$$L = \frac{1}{2} T^{ij}_{ab}(q^k_c)\, \dot{q}^i_a \dot{q}^j_b + H^i_a(q^k_c) q^i_a. \tag{15}$$

One may interpret T^{ij}_{ab} as the metric and H^i_a as a linear connection or a vector potential. The connection H^i_a has been obtained explicitly in terms of the self-dual configurations.[11] However no satisfactory answer for T^{ij}_{ab} has been found in spite of several attempts.[11,45] This situation contrasts to the self-dual Maxwell-Higgs case.[46]

When there is a uniform background charge, moduli space approximation becomes more interesting. Again H^i_a is known but T^{ij}_{ab} is not.[16,15] If moduli space approximation is reasonable, a single vortex moves a circular motion on this background due to the Magnus force. In some range of the parameter space, the rest mass of vortices becomes negative. (The total energy of a pair of vortex and antivortex is positive and so the system is stable.) I do not have any clue for the method to calculate the kinetic mass in this case. On the other hand the energy difference of the Landau levels after quantization can be larger than the rest mass of some elementary neutral quanta in the broken phase. This contradicts the spirit of moduli space approximation where we expect only zero modes to be excited at

low energy. This makes me to wonder whether moduli space approximation is good at all here.

6. Concluding Remarks

The relativistic self-dual Chern-Simons systems come with many flavors. Their vacuum and soliton structures are rich and diverse. They have been a playing ground for testing and sharpening our understanding of quantum field theory of anyons and solitons. I have discussed many ideas and questions related the Chern-Simons systems.

There are also many interesting topics I have not discussed at all: the potential force between vortices away from self-duality, the finite temperature correction to the Chern-Simons coefficient in the symmetric phase, the theories on compact Riemann surfaces, quantum Hall effects and boundary states, semi-local solitons, supergravity models behind the self-dual models, etc. I believe that there are still more surprises and insights to be discovered in this field.

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